

**Prepared by
Mr /Ahmed Omar**

Geometric Concepts

III The line segment:

It is a set of points consisting of two distinct points and all points between them when we join them by a ruler.

- denoted by \overline{AB} or \overline{BA}



- The line segment has two ends and a length.
- The length of \overline{AB} is denoted by AB or BA .

② The ray:

It is a line segment extended from only one of its terminals without limit.

- denoted by \vec{AB}



- \vec{AB} differs \vec{BA}
- The ray has a starting point and it has no end point, therefore it has no length.

13) The straight line:

If we extend the line segment in both directions infinitely, we will get a straight line.

- denoted by \overleftrightarrow{AB} or \overleftrightarrow{BA}



- The straight line doesn't have a starting point or an end point, hence the straight line has no length.

Remark. $\overline{AB} \subset \overrightarrow{AB} \subset \overrightarrow{AB}^{\text{no end}}$

④ The plane:

It is a flat and unbounded surface, and it is extended without limit in all directions.

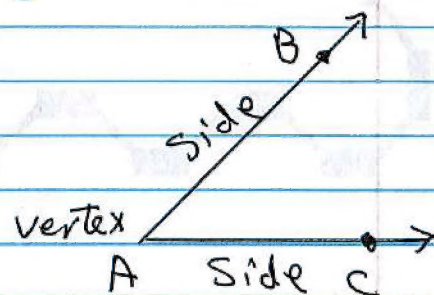
-

5) The angle:

It is the union of two rays with the same starting point.

$i \cdot e$

$$\vec{AB} \cup \vec{AC} = \angle CAB$$
$$\text{or } \angle BAC$$
$$\text{or } \angle A$$



16] Measurement of The angle:

- It is the number expressing the amount of happened divergence between the two sides.
- The angle is measured using degree unit which is denoted by ($^{\circ}$)

where

$$1^\circ = 60', \quad 1' = 60''$$

1° degree, $1'$ minute, $1''$ second

The types of angles:

The angles are classified according to their measures as follows:

1) Zero angle

Its measure $= 0^\circ$

its sides are coincident

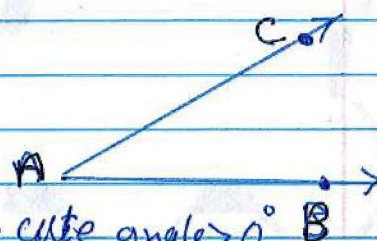
$$m\angle BAC = 0^\circ$$



2) Acute angle:

Its measure more than 0° and less than 90°

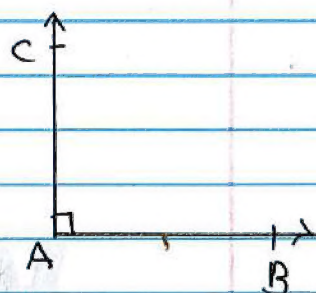
i.e. $90^\circ > \text{measure of acute angle} > 0^\circ$



3) Right angle:

Its measure $= 90^\circ$

$$m\angle BAC = 90^\circ$$

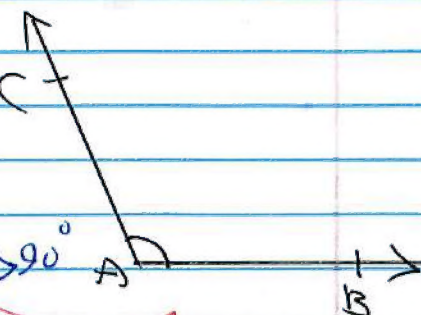


4) Obtuse angle:

Its measure is more than 90° and less than 180° .

i.e.

$180^\circ > \text{measure of obtuse angle} > 90^\circ$



5) Straight angle:

Its measure $= 180^\circ$

i.e.

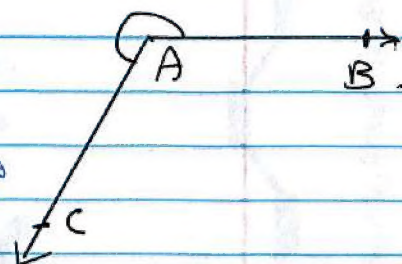
$$m\angle BAC = 180^\circ$$



6) Reflex angle:

Its measure is more than 180°
and less than 360°

i.e $180^\circ < m(\angle BAC) < 360^\circ$



Remark: In the opposite figure:

$$m(\angle BAC) + m(\text{reflex } \angle BAC) = 360$$

i.e

$$m(\text{reflex } \angle BAC) = 360 - m(\angle BAC)$$

Ex: Mention the type of each of the angles which their measures are as follows:

① $45^\circ \Rightarrow$ It is an acute angle

② $96^\circ \Rightarrow$ It is an obtuse angle

③ $90^\circ \Rightarrow$ It is a right angle

④ $160^\circ \Rightarrow$ It is an obtuse angle

⑤ $240^\circ \Rightarrow$ It is a reflex angle

⑥ $125^\circ 30' \Rightarrow$ It is an obtuse angle

⑦ $180^\circ \Rightarrow$ It is a straight angle

⑧ $89^\circ 60' \Rightarrow 89^\circ 60' = 90^\circ$ It is a right angle

⑨ $179^\circ 59' 60'' \Rightarrow 179^\circ 59' 60'' = 180^\circ$ It is a straight

⑩ $89^\circ 62' \Rightarrow 89^\circ 62' = 90^\circ 2'$ It is an obtuse

Supplementary angles:

Two angles are said to be supplementary if the sum of their measures is 180° .

Remarks:

- ① The supplement of an obtuse angle is an acute angle
- ② the supplement of a right angle is a right angle
- ③ the supplement of a zero angle is a straight angle
- ④ the supplements of the same angle (or equal angles in measure) are equal in measure.

i.e. If $\angle A$ supplements $\angle B$ and $\angle C$ supplements $\angle B$, then $m(\angle A) = m(\angle C)$.

Ex1 Complete the following table:

measure of the angle	30	90	40°	acute
measure of its complement	60°	0°	50°	acute

- the complement of an angle = 90° - the measure of this angle

Ex2 Complete the following table:

measure of the angle	30°	110°	0°	90°	acute
measure of its supplement	150°	70°	180°	90°	obtuse

The supplement of an angle =

= 180° - the measure of this angle

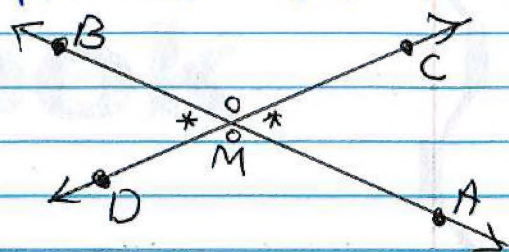
Vertically opposite angles (V.O.A):

If two straight lines intersect, then the measures of each two vertically opposite angles are equal.

In the opposite figure:

If $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$ then,

$$\begin{aligned} m\angle AME &= m\angle BMD & (\text{V.O.A}) \\ \text{also, } m\angle CMB &= m\angle AMD & (\text{V.O.A}) \end{aligned}$$

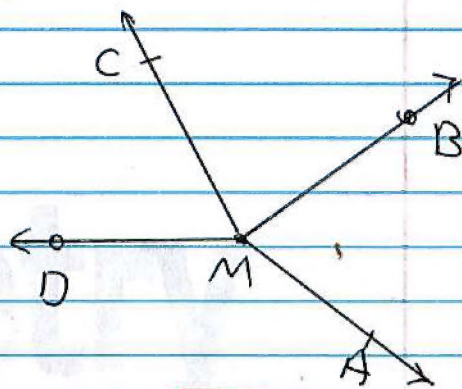


Accumulative angles at a point:

The sum of the measures of the accumulative angles at a point is 360° .

In the opposite figure:

$$\begin{aligned} m\angle AMB + m\angle BMC + m\angle CMD \\ + m\angle DMA = 360^\circ \end{aligned}$$



The angle bisector:

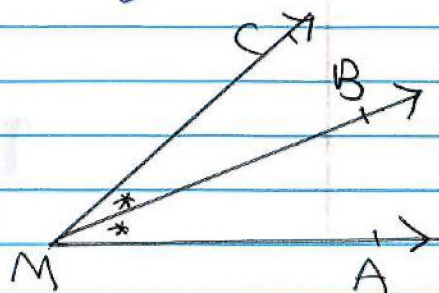
It is the ray that divides the angle into two halves (two equal angles in measure).

In the opposite figure:

\overrightarrow{MB} bisects $\angle AMC$

i.e.

$$m\angle AMB = m\angle BMC = \frac{1}{2} m\angle AMC$$



if $\overline{MC} \perp \overleftrightarrow{AB}$

then: $m(\angle AMB) = 180^\circ$ "straight angle"

Therefore, $M(\angle CMB) = 90^\circ$

Diagram illustrating a right angle (90°) formed by a vertical line segment CB and a horizontal line segment AB . A ray BD is drawn from vertex B , dividing the right angle into two adjacent angles: $\angle CBD = x^\circ$ and $\angle ABD = 60^\circ$. The text "adj angle" is written next to the diagram.

$$x = 180 - (15 + 15 + 90)$$

~~$$x = 360 - (50 + 90 + 100)$$~~

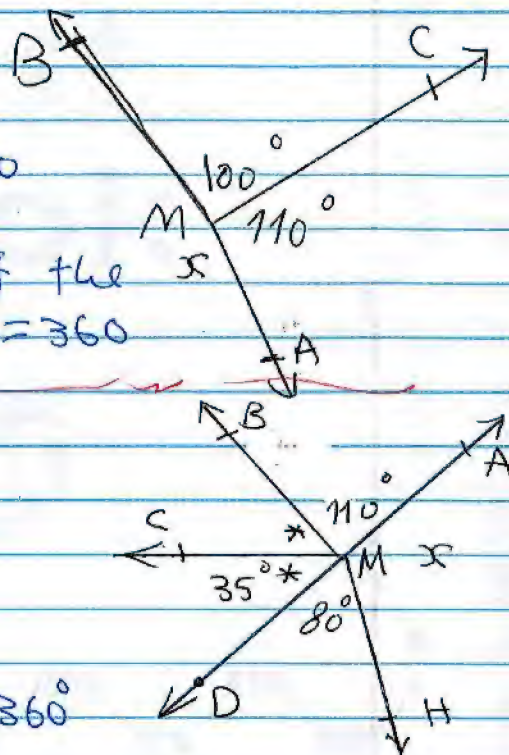
the
 $t = 360^\circ$

Solution

Because the sum of measures of the accumulative angles at a point = 360

$$= 360 - 260 = 100^\circ$$

Because the sum of measures of the accumulative angles at a point = 360°


$$AD \cap BH = \{M\}$$
$$m(\angle CMD) = 28^\circ \text{ and } m(\angle BMC) = 115^\circ$$

find the measures of the angles marked by (?)

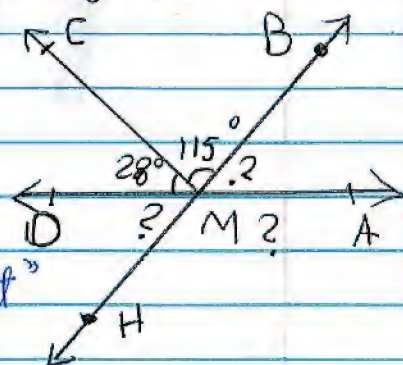
Solution

$$\begin{aligned} \bullet \quad m(\angle AMB) &= 180^\circ - (115^\circ + 28^\circ) \\ &= 180 - 143 = 37^\circ \end{aligned}$$

Because $m(\angle AMD) = 180^\circ$ "straight angle"

• $m(\angle DMH) = m(\angle BMA) = 37^\circ$ (v.o.A)

- $m(\angle AMH) = m(\angle BMD) = 143$ (V.O.A)

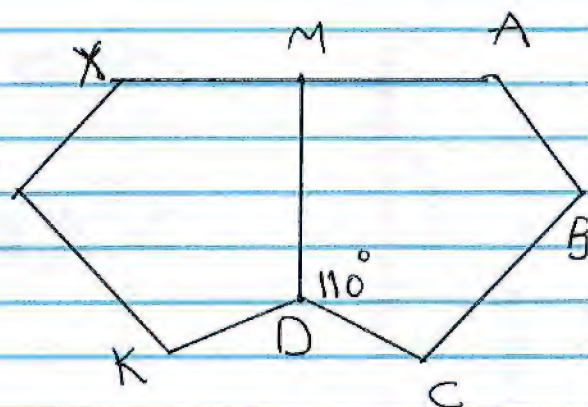


In the opposite figure:

If the figure

$ABCDM \equiv$ the figure $XYKDM$

Complete



① $AB = \dots$, $CD = \dots$

② $BC = \dots$, $XM = \dots$

③ $m(\angle A) = m(\angle \dots)$, $m(\angle B) = m(\angle \dots)$

④ $m(\angle C) = m(\angle \dots)$, $m(\angle CDM) = m(\angle \dots)$

⑤ the axis of symmetry of figure $ABCDKXY$ is...

⑥ $m(\angle AMD) = \dots^\circ$

⑦ $m(\angle CDK) = \dots^\circ$

⑧ \overline{MD} is called.

Solution

we deduce from the Congruence of the two polygons

① $AB = XY$, $CD = KD$

② $BC = YK$, $XM = AM$

③ $m(\angle A) = m(\angle X)$, $m(\angle B) = m(\angle Y)$

④ $m(\angle C) = m(\angle K)$, $m(\angle CDM) = m(\angle KDM)$

⑤ The axis of symmetry is \overline{MD}

⑥ $m(\angle AMD) = \frac{180^\circ}{2} = 90^\circ$ So we deduce that $\overline{MD} \perp \overline{AX}$

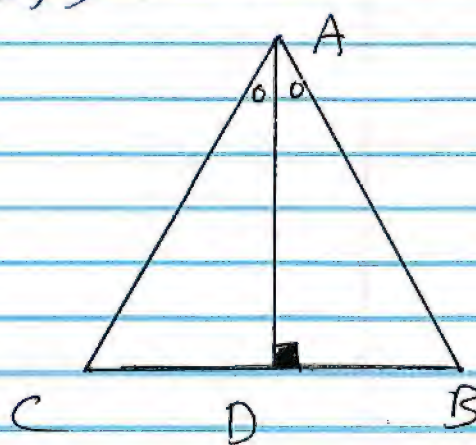
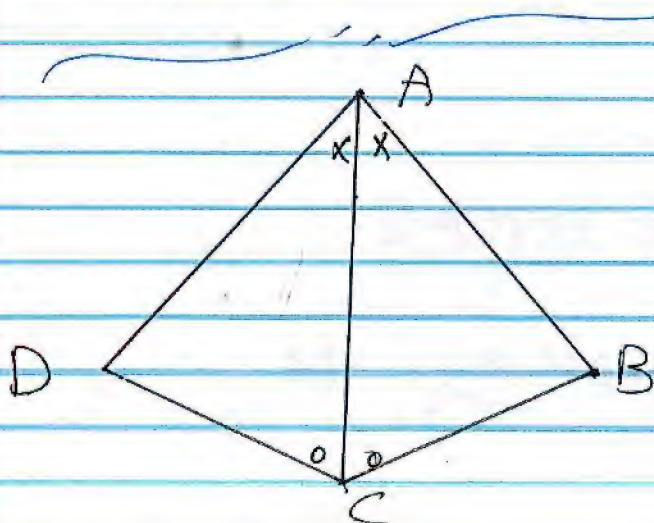
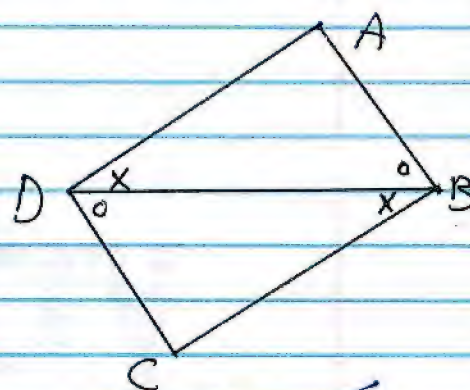
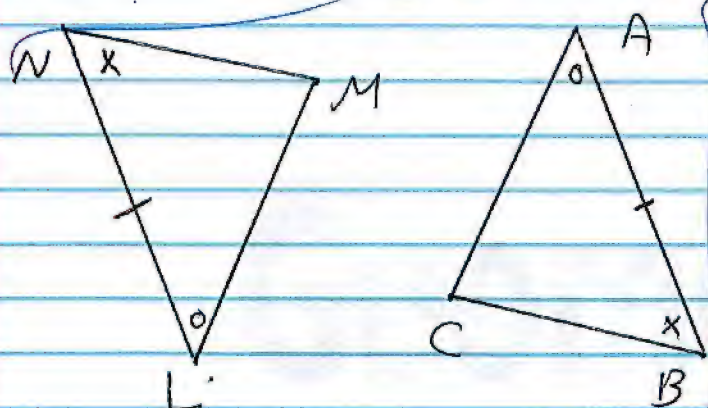
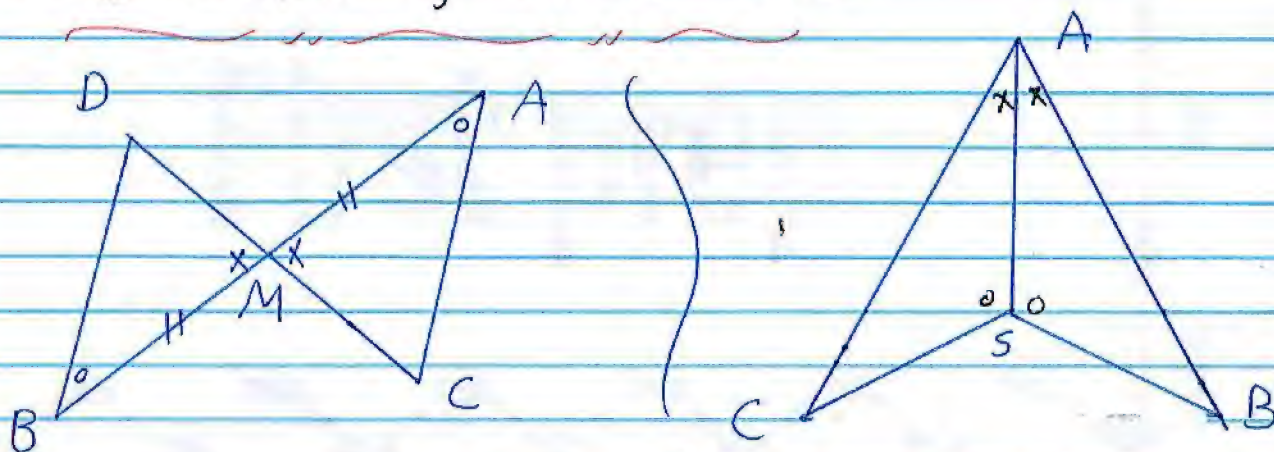
⑦ $m(\angle CDK) = 360 - (110 + 110) = 360 - 220 = 140^\circ$

⑧ \overline{MD} is called a Common Side

The second case of congruence of two triangles
(two angles and one side A.S.A)

Two triangles are Congruent if two angles and the side drawn between their vertices of one triangle are Congruent to the Corresponding parts of other triangle.

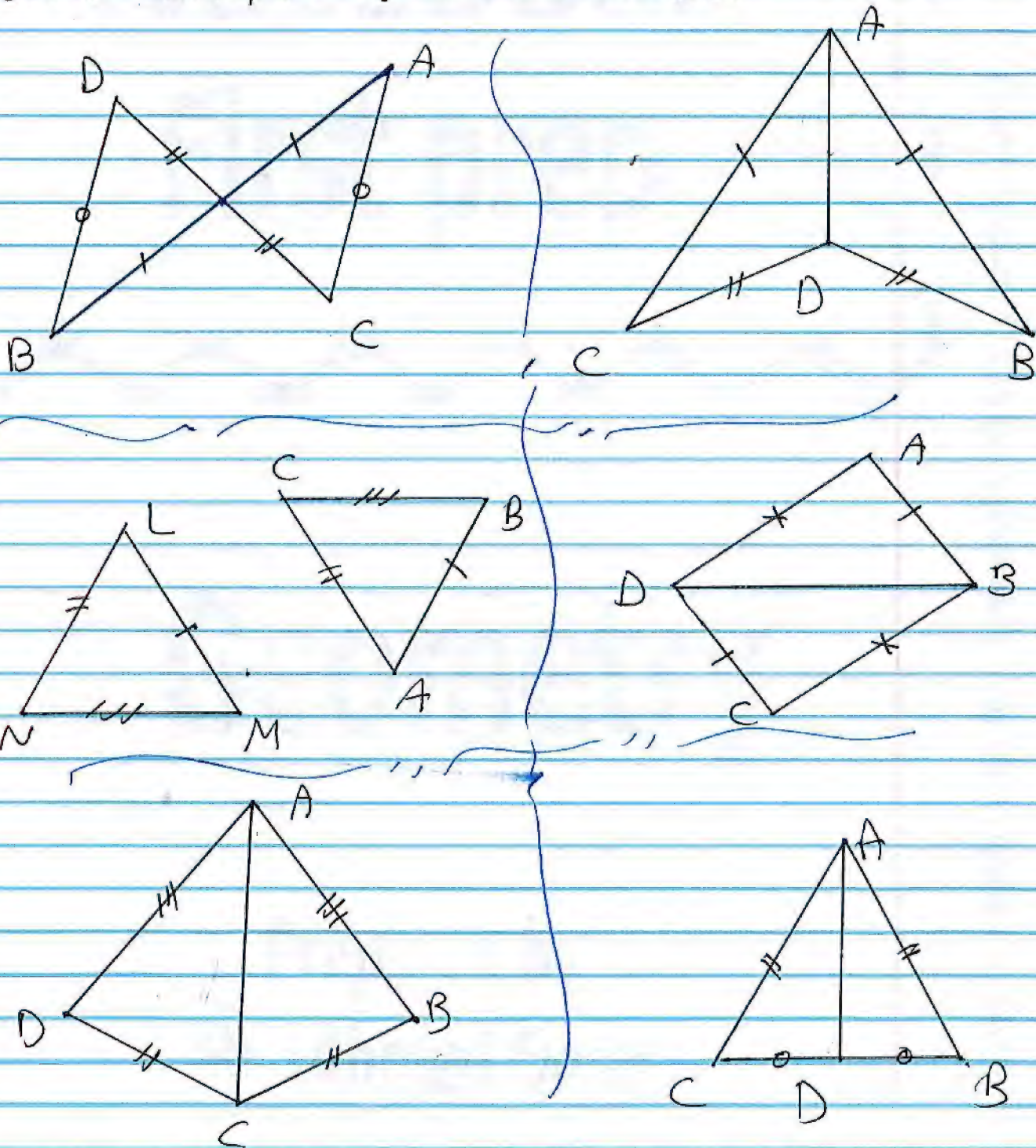
Some examples of second case:



The third Case of Congruence of two triangles
(Three sides S.S.S.)

Two triangles are congruent if each side of one triangle is congruent to the corresponding side of the other triangle

Some examples of the third case:



Ex: In the opposite figure:

$$BD = DC, m(\angle ADB) = m(\angle ADC)$$

Explain why does \vec{AD} bisect angle A?

Solution

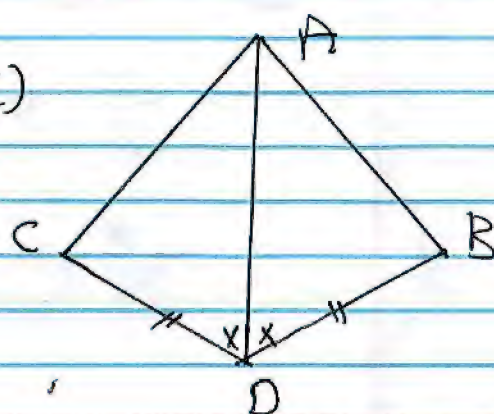
$$\triangle ABD \cong \triangle ACD$$

"two sides and included angle" (S.A.S)

we deduce from the congruence that:

$$m(\angle BAD) = m(\angle CAD)$$

i.e: \vec{AD} bisects $\angle A$



In the opposite figure:

$$m(\angle B) = m(\angle D) = 90^\circ$$

$$HD = AB, BC = 5 \text{ cm}$$

$$AC = AH$$

Find: AD , $m(\angle AHD)$

with showing the steps of solution

Solution

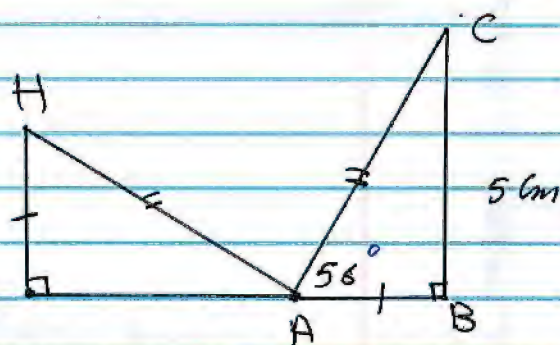
$$\begin{cases} m(\angle B) = m(\angle D) \\ HD = AB \quad (\text{side}) \\ AC = AH \quad (\text{Hypotenuse}) \end{cases}$$

then $\triangle ABC \cong \triangle HDA$ (R.H.S)

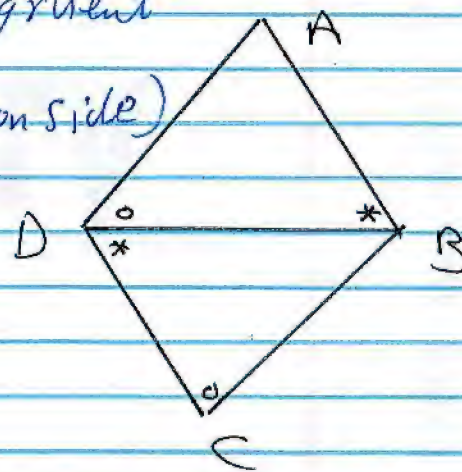
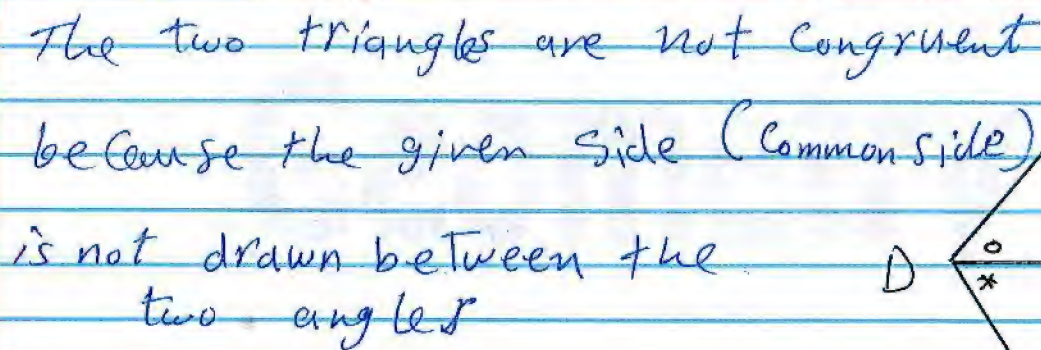
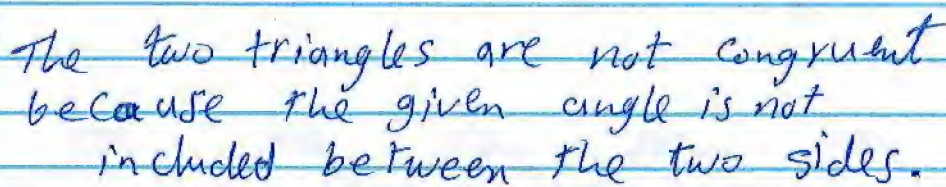
and we deduce from the congruence that:

$$AD = BC = 5 \text{ cm}$$

$$, m(\angle AHD) = m(\angle CAB) = 56^\circ$$



① The two triangles are not congruent because data is not enough.



Relation between pairs of angles formed from two parallel straight lines and a transversal to them:

If a straight line intersects two parallel straight lines, then

- ① Each two alternate angles are equal in measure.
- ② Each two corresponding angles are equal in measure.
- ③ Each two interior angles in the same side of the transversal are supplementary.

The Condition of parallelism of two straight lines:

The two straight lines are parallel if a third straight line intersects them and one of the following cases is satisfied:

- ① Two alternate angles have the same measure.
- ② Two corresponding angles have the same measure.
- ③ Two angles in the same side of the transversal are supplementary.

Geometric facts:-

- ① The perpendicular to one of two coplaner parallel straight lines is perpendicular to the other
- ② If two coplaner straight lines are perpendicular to a third one, then the two straight lines are parallel.
- ③ If two straight lines are parallel to a third, then these two straight lines are parallel.
- ④ If parallel straight lines divide a straight line into segments of equal length, then they divide any other straight line into segments of equal lengths.

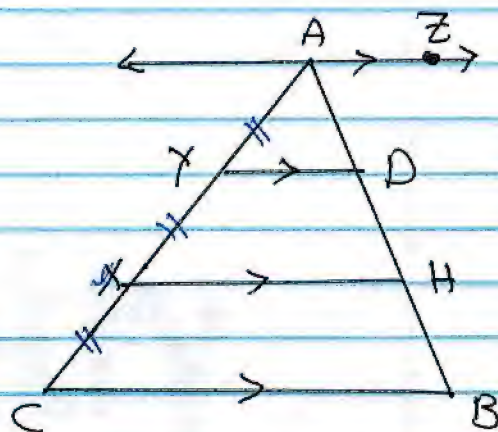
In the opposite figure:

$$\overleftrightarrow{AZ} \parallel \overleftrightarrow{YD} \parallel \overleftrightarrow{XH} \parallel \overleftrightarrow{CB}$$

2) \overleftrightarrow{AB} and \overleftrightarrow{AC} are their Transversals

$$AY = YX = XC$$

Then, $AD = DH = HB$



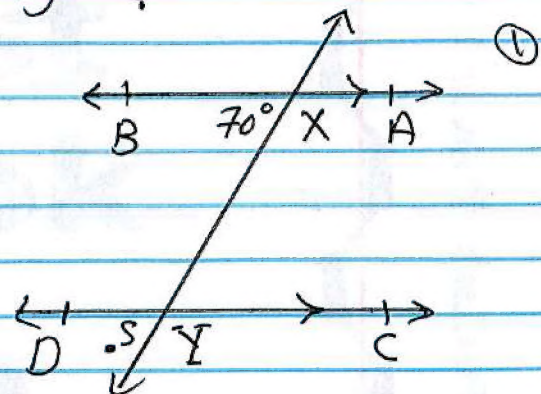
Ex: In each of the following figures, find the measure of the angle which is marked by "?"

Solution

$$m(\angle DYH) = m(\angle BXY) = 70^\circ$$

(alternate angles)

Because $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and \overleftrightarrow{XY} is transversal

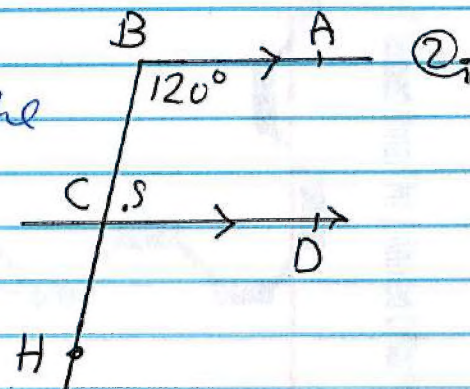


$$m(\angle B) + m(\angle BCD) = 180$$

interior angles in the same side of the transversal

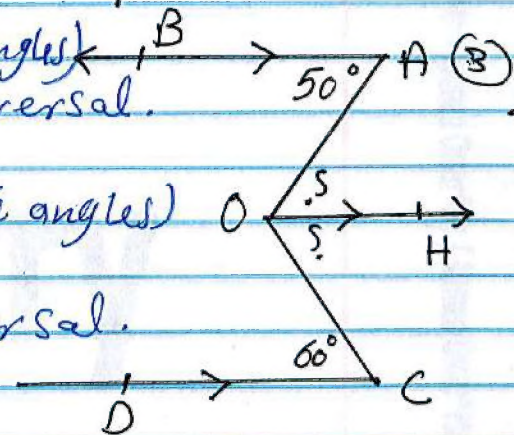
then:

$$m(\angle BCD) = 180 - 120 = 60^\circ$$



- $m(\angle AOH) = m(\angle A) = 50^\circ$ (alternate angles)
Because $\overleftrightarrow{AB} \parallel \overleftrightarrow{OH}$, \overleftrightarrow{AO} is the transversal.

- $m(\angle HOC) = m(\angle C) = 60$ (alternate angles)
Because $\overleftrightarrow{OH} \parallel \overleftrightarrow{CD}$, \overleftrightarrow{OC} is a transversal.



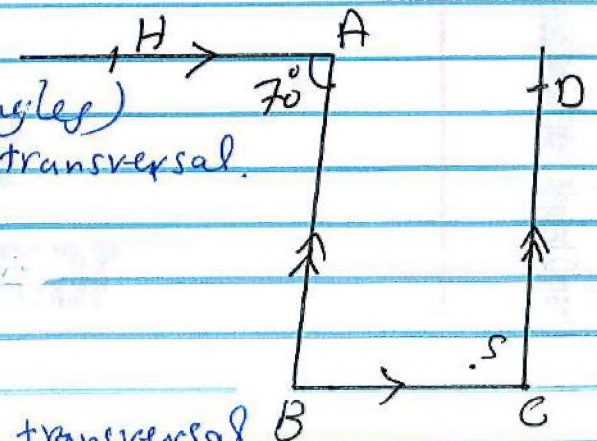
- $m(\angle B) = m(\angle A) = 70^\circ$
(alternate angles)
Because $\overleftrightarrow{AH} \parallel \overleftrightarrow{BC}$, \overleftrightarrow{AB} is a transversal.

$$m(\angle B) + m(\angle C) = 180^\circ$$

(interior angles)

$$m(\angle C) = 180^\circ - 70^\circ = 110^\circ$$

Because $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, \overleftrightarrow{BC} the transversal



Ex: In the opposite figure

$$\vec{AD} \parallel \vec{CB}, m(\angle DAH) = 70^\circ$$

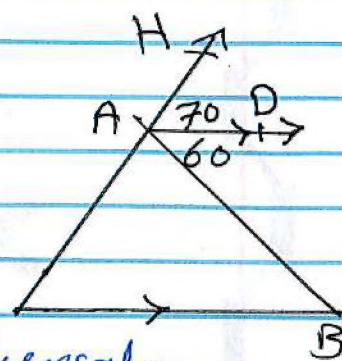
Find: $m(\angle B)$, $m(\angle C)$, $m(\angle BAC)$

Solution:

$m(\angle C) = m(\angle HAD) = 70^\circ$ corresponding
Because, $\vec{AD} \parallel \vec{BC}$, \vec{AC} the transversal.

$m(\angle B) = m(\angle DAB) = 60^\circ$ alternate angles.
Because, $\vec{AD} \parallel \vec{BC}$, \vec{AB} the transversal.

$$m(\angle BAC) = 180 - (60 + 70) = 180 - 130 = 50^\circ$$



In the opposite figure:

$$\vec{BA} \parallel \vec{CD}, m(\angle BCA) = 150^\circ, m(\angle DCH) = 90^\circ$$

Find: $m(\angle B)$.

Solution:

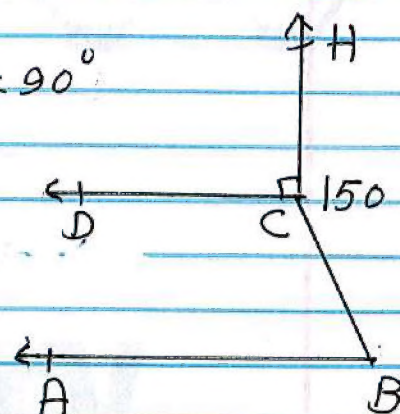
$$\begin{aligned} m(\angle DCB) &= 360 - (90 + 150) \\ &= 360 - 240 \\ &= 120^\circ \end{aligned}$$

Because,

The sum of measures of the accumulative angles at a point $= 360^\circ$

$$m(\angle B) = 180 - 120 = 60^\circ \text{ (interior angles)}$$

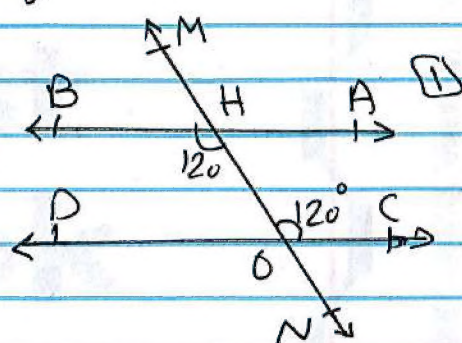
Because, $\vec{CD} \parallel \vec{BA}$, \vec{BC} the transversal.



In each of the following, show why $\overrightarrow{AB} \parallel \overrightarrow{CD}$

Solution $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

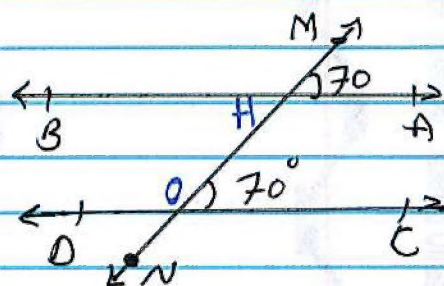
Because $m(\angle BHO) = m(\angle HOC) = 120^\circ$
and they are alternate angles



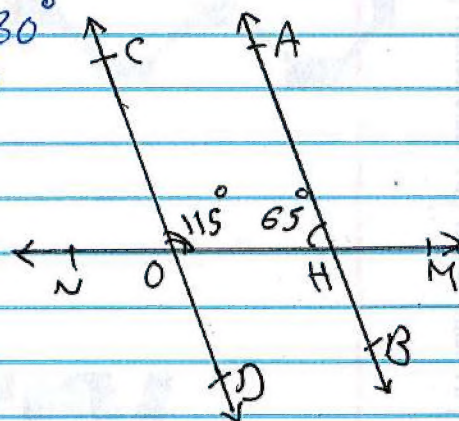
$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

Because $m(\angle AHM) = m(\angle HOC) = 70^\circ$

and they are corresponding angles.



$m\angle AHO + m\angle COH = 115 + 65 = 180^\circ$
 and they are interior angles in
 one side of the transversal,
 therefore; $\vec{AB} \parallel \vec{CD}$



$$m(\angle BAC) = 2 \times 50 = 100^\circ$$

$$m(\angle BAC) + m(\angle C) = 100 + 80 = 180$$

and they are interior angles in one side of the transversal.

Therefore, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

